

Modern Portfolio Theory and Optimal Investment
Allocation Decisions of an Insurance Company

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I. Overview

The principal goal of this paper is to demonstrate the utility of modern portfolio theory (MPT) in the investment allocation decisions of an insurance company. The demonstration utilizes more than the traditional investment portfolio optimization, however, because it considers the qualities of *both* the asset (investment) side *and* the liability side of an insurer's balance sheet in finding the optimal allocation of investments. A second, related goal of the paper is to address the liability side of the balance sheet, and respond to commentators' suggestions that an insurance company may be financially better off by relying wholly on MPT and investments to mitigate underwriting risk, as opposed to using reinsurance.

The results of the tests herein demonstrate an MPT model that increasingly adds more information regarding an insurer's total portfolio of investments *and* liabilities provides superior risk-return characteristics than an investment and liability portfolio that are balanced without consideration of one another. What is more, the tests herein demonstrate the return on the insurer's total portfolio of investments and liabilities improves still further with the introduction of traditional quota share (proportional) reinsurance.

a. Summary of the Paper

The paper begins by numerically defining a fictitious, multi-line property and casualty insurance company based on industry-wide data. For purposes of applying MPT later in the analysis, the company's underwriting book is sometimes referred to as a liability (a use of funds). The company's book of business includes lines of varying "tails" or durations. The investment side provides the funds to support these liabilities. The paper uses a set of mutual funds and indexes to proxy the type and duration of investments in which a traditional property casualty insurer might invest. The investments and liabilities are then "paired" based on their durations to ensure investment liquidity when the company expects liabilities to come due for payment.

Once the paper specifies the data, the mean-variance model is set forth with restrictions on short sales, cash flow matching, etc. and a Base Case scenario of the insurer's investment and liability allocations is created. All subsequent scenarios or cases are compared to this base case to determine whether application of the MPT methodology and reinsurance yield improvements to the insurer's total return on its portfolio.

The Base Case uses a naïve, equally weighted investment portfolio that is allocated among individual investments without considering the insurer's liability or

underwriting decisions. The allocation of the weights to each line of business in the underwriting portfolio is based on industry averages, and remains the same in all cases (it is never optimized though it could be). The mean return and standard deviation of the insurer's total portfolio of investments and liabilities is then computed to serve as the starting point for comparisons.

Case 2 continues to consider separately investment and liability allocation decisions;. The MPT methodology is applied to allocate the weights of the component parts of the investment side. A total portfolio return and sigma are calculated and compared to the Base Case.

Case 3 applies MPT to the investment side allocations; however, the application of MPT *includes* consideration of liability characteristics in determining the optimal allocation of investments. It is this application of MPT to both assets and liabilities that separates the present analysis from the traditional use of the methodology.

Finally, in Case 4, the previous case is duplicated though it uses liability data subjected to a quota share reinsurance program. The paper concludes with a comparison of the risk-return and investment allocation characteristics of each test case.

b. Modern Portfolio Theory¹

Harry Markowitz introduced the world to modern portfolio theory with the publication of his paper "Portfolio Selection" in the Journal of Finance in 1959. Thirty-eight years later, he shared a Nobel Prize with Merton Miller and William Sharpe for what has become a broad theory for portfolio selection and corporate finance. Despite the extreme usefulness of MPT and the Nobel prize associated with it, financial institutions did not fully embrace it in a significant way until the late 1980's. Today, MPT is the guiding principle behind portfolio allocation decisions for mutual funds, pension funds, and virtually any entity seeking to maximize investment portfolio returns and minimize risk.

MPT explores how risk-averse investors can construct optimal portfolios by considering the trade-off between market risk and expected returns. The theory quantifies the benefits of diversification. Out of a universe of risky investments, an investor can construct an efficient frontier of optimal portfolios. Each portfolio on the efficient frontier offers the maximum possible expected return for a given level of risk. Investors therefore should hold one of the optimal portfolios on the efficient frontier and adjust total market risk by leveraging or de-leveraging that portfolio with positions in a risk-free investment such as government bonds.

The MPT model is a single-period approach. It assumes an investor has a given initial endowment to invest. The investor will hold the investment for a specific length of time. At the end of that period, the investor will decide to keep the investments invested (or re-invest them) or consume them, or employ a combination of both. This is otherwise known as a fixed mix or a buy-and-hold strategy. If the investor decides to keep the investments invested, he will re-evaluate the mix of investments in the portfolio, find the new efficient frontier, and rebalance his portfolio to ensure it remains optimized.

While institutional funds use MPT regularly, insurance companies tend to embrace it less.² This is because, unlike an institutional fund, an insurance company is essentially a portfolio of *both* investments *and* liabilities. Moreover, heavy regulation of the insurance industry places far more constraint on an insurer's investment decisions than that experienced by mutual or other funds. Despite the foregoing, and as explained herein, the MPT model definitely has a place in the insurer's portfolio decision-making process.

c. Fictitious Industry Insurance Company

The underwriting experience of the company that is the subject of this paper was constructed from aggregate US property and casualty industry data. Our analysis is limited, however, to the company's investment and liability or underwriting portfolios. This narrow view is all that is necessary to prove the paper's point regarding MPT, allowing for simplification of the inquiry. As a result, the effect of MPT on policyholder's surplus or owners' equity is not considered in the analysis (though it could be considered).

Property And Casualty Insurance Generally³

The property and casualty insurance industry plays a vital role in the economy by spreading and absorbing risk, converting a small chance of disastrous loss (a ruinous claim) to the certainty of a relatively small loss (the premium). In most enterprises, and in daily living, the certainty of a small loss is greatly preferable to a slight chance that one could lose everything through the vagaries of fire, weather and other contingencies. Without this ability to replace the unknown with the known, it is almost certain that the United States and many other economies could never have developed to their present sizes, sophistication and complexity: lenders and shareholders would continuously be exposed to the possibility that all could be lost through some catastrophic event, doubtless constraining the investment of capital.

The mechanism of insurance is simple enough. For example, we know that the risk of fire on a given house is very small. But we may also know that in a community with a thousand homes it is almost certain that at least two will be destroyed by fire in a particular year. If each home is worth \$100,000 then we expect the losses in a year to total about \$200,000. Since we have a thousand homes to share the risk, if we receive a premium of \$200 for each home we will have collected enough to pay for the two total losses that are expected to occur. Of course our simple insurance company will require some money to cover the costs of marketing and distributing its product and to pay for overhead and administration, so in fact the premium will have to be somewhat higher than \$200, but the principle is the same. A traditional saying in the industry is that “the premiums of the many shall pay for the losses of the few.”

The volatile nature of the property and casualty insurance business is also illustrated by the foregoing example. We can see that if there happen to be three total losses the insurer will in fact have lost \$100,000. Or if there is a serious windstorm or earthquake, the insurer itself could be financially crippled and unable to pay its claims.

The Role of Reinsurance

To protect against this eventuality the insurer maintains two lines of defense. First, it shares its risks with reinsurers: reinsurers are insurers that specialize in the assumption of risks from other insurers. Reinsurance is essentially insurance for insurance companies. Reinsurers tend to be extremely large with operations on an international basis. By broadly diversifying their operations they reduce their exposure to loss. In terms of the above example, the chance of a catastrophic loss, relative to the amount of premiums collected, is much smaller if you are insuring a portion of one thousand neighborhoods in ten countries than if you are insuring one thousand homes in one neighborhood.

Reinsurance can be structured so that the reinsurer will share in a portion of every loss, or so that the reinsurer will absorb all losses once they exceed a certain level in the aggregate or once they exceed a certain level from one event such as a storm or earthquake, and in other ways as well. Reinsurance can also be applied to an entire portfolio of business or to a particular property such as an offshore oilrig or major building. Insurers usually structure their reinsurance so that coverage is spread amongst a number of reinsurers. The combinations are almost endless and an important aspect of an insurer’s expertise is the ability to structure its reinsurance arrangements so as to represent an appropriate balance of protection and cost.⁴

The Importance of Policyholders Surplus

Another practice followed by property and casualty insurers to help ensure that they will be able to pay all future claims, is to maintain relatively higher levels of capital (i.e. excess of investments over liabilities) than other types of financial institutions where future financial obligations can be predicted with greater certainty. Additionally, funds must be invested in quite a conservative manner because of the high level of business risk being assumed through the insurance function. When a large claim is incurred, the insurer must be able to get funds into the hands of the policyholder without undue delay.

Insurance versus Other Financial Services

The property and casualty insurance industry is quite different from other types of financial services business. Unlike banks and life insurers, property and casualty insurers are not in the financial intermediation business. They are in the business of intermediating risk rather than financial investments, and as a part of this function they must take on a good deal of business risk themselves. This impacts the business in a number of fundamental ways, including investment strategy and capital allocation. Additionally, when one looks at the financial performance of the industry over any period of five years or more, one cannot help but be struck by the high degree of cyclicity in the results.

Property and casualty insurers are really involved in two businesses: (1) insurance, where the main functions are the assessment of risk, the setting of appropriate premiums to compensate for the risks being assumed and the paying of claims; and (2) investing, where premiums must be invested pending the need for payment of claims and other purposes.

Typical risks covered by property and casualty insurers include the risk of loss or damage to property (e.g. a typical homeowner's policy), the risk of being held liable for causing loss or damage to others (e.g. the liability portion of an automobile insurance policy) and other more specialized risk situations.

d. Should an Insurer Rely on Optimized Investments or Reinsurance?

It is important to highlight from the above summary the conservative nature of an insurer's investments, and the role of preservation and maximization of policyholders surplus. Both these points are critical to the insurer's ability to make timely and complete payments to its policyholders on their loss claims.

Application of MPT can play a role in the insurer making proper allocation decisions on the investment side to ensure timely availability of the right amount of funds to pay losses. Reinsurance is a tool for controlling risk on the liability side. Both MPT and reinsurance, therefore, can mitigate some of the insurer's problems in estimating the correct price for its products, and soften the blow of unforeseen events that affect business adversely.

The insurer's need for risk mitigation tools (such as MPT and reinsurance) is critical. This is because, when an insurer prices its products it takes an actuarial "guess" at the real cost of the product, however, the company typically will not know the true cost of the product until years later, when most claims have been filed against the policy, and the total amount of losses and related expenses are finally known. Many events, expected and unexpected, can occur from the time the insurer receives the premium for the insurance and invests it to the time it pays out the last dollar in losses. For example, on the underwriting side, losses could be far higher than estimated due to an unexpected catastrophe. On the investment side, returns on investments could be far lower than estimated due to an unexpected market shock or dislocation. Either way, the pricing risk exists in that the insurer will not receive enough premium or return on investment of the premium to cover losses, and the insurer will suffer a loss in its total portfolio of investments and liabilities.

Some commentators have opined that an insurer may rely almost entirely on an optimized investment portfolio and forego buying reinsurance coverage in order to mitigate unforeseen perils and market events that affect the company's bottom line.⁵ That is, these commentators suggest an insurer may, in the face of unattractive reinsurance arrangements, be better off securitizing its insurance risks within its optimized investment portfolio, rather than purchasing reinsurance. As demonstrated below, however, reinsurance typically has a salutary effect on the investment as well as the liability side of the ceding insurer's balance sheet.

II. The Data

This analysis simulates a property and casualty insurance company by using industry aggregate underwriting data for fifteen lines of business, and eighteen mutual funds and index funds to simulate the insurer's investments. The items in each of these portfolios are then "matched" by comparing their durations. A mean-variance model is then applied to this data and the resulting total portfolio return and sigma calculated.

a. Underwriting Data

This data addresses the lines of business that our property and casualty insurer writes. The insurer assumes risk of loss from policyholders in these lines in return for an insurance premium. This premium is the source of the insurer's investments on the investment side of its balance sheet. This time series data covers the years 1989 through 1999 and includes by-line industry aggregate dollar figures for net written and earned premium, loss ratios, loss adjustment expense (LAE) ratios and underwriting expense ratios. The data includes only pure underwriting numbers, and for purposes of simplification, the data omits corporate and premium taxes. Application of MPT to an actual company's data should best consider after-tax data due to the taxable nature of the business. The source of the data was Best's Aggregates and Averages for 2000 and 1990.

Multiplying the loss ratios by earned premium and the underwriting expense ratio by written premium yielded dollar amounts for the ratios that were then used to calculate by-line underwriting results or returns. For the purposes of this endeavor, by-line underwriting return is defined as

$$(\text{Earned Premium} - \text{loss payments} - \text{LAE} - \text{Underwriting Expenses}) / \text{Earned Premium}$$

If the product of this equation is positive, the line has earned an underwriting profit; if it is negative, the line has incurred an underwriting loss. In order to simplify the analysis, underwriting results do not explicitly account for the time value of earnings on invested premium. This information would increase underwriting returns, especially for long-tail (i.e., long duration) lines of business.

Line of Business	Underwriting Return 1989 to 1999	Standard Deviation 1989 to 1999
Fire	-3.91%	5.67%
Allied (includes Earthquake)	-17.67%	43.64%
Homeowners Multiple Peril	-17.50%	14.22%
Ocean Marine	-8.91%	9.52%
Inland Marine	1.62%	3.39%
Workers Compensation	-4.68%	8.95%
Other Liability	-20.42%	11.84%
Medical Malpractice	-4.71%	11.15%
Auto Liability Private	-7.11%	6.78%
Auto Liability Commercial	-12.56%	3.48%
Auto Physical Damage Private	5.24%	4.41%
Auto Physical Damage Commercial	3.35%	9.60%
Fidelity	15.19%	12.64%
Surety (Incl Fin. Gty Since 1986)	16.66%	8.35%
Burglary/Theft	34.65%	6.36%

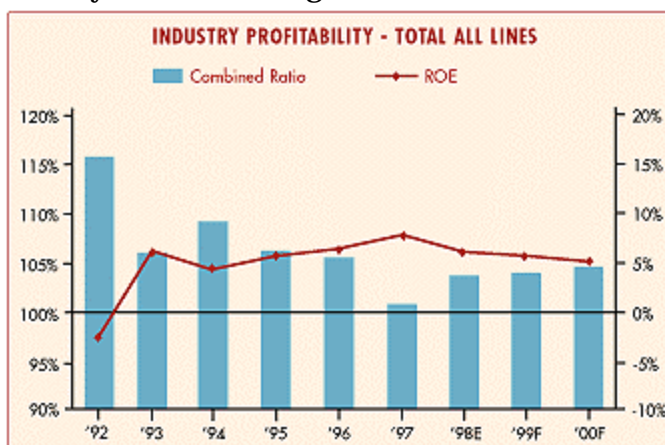
The average return over the ten-year period for all fifteen lines of business considered was -1.38% with a standard deviation of 10.67%, a minimum of -20.42% (the “other liability” line) and a maximum of 34.65% (burglary/theft line). As is common in property and casualty insurance, nine of the fifteen lines incurred an underwriting loss. The complete underwriting data and calculations are located in Appendices 1 and 2.

Why Did So Many Lines Incur Underwriting Losses?

The answer to this question is simply that the US property and casualty insurance industry as a whole suffered an unusually large amount of losses during the period considered (1989 – 1999). In fact, the ten most costly (natural) catastrophes ever to hit the United States did so between 1989 and 1999, including Hurricane Andrew in 1992, and the Northridge California earthquake in 1994.⁶ What is more, the industry entered a period of stiff competition and related underpricing during the same time frame that depressed the amount of premium written and earned by insurers. The result was negative cash

flows from the property and casualty industry’s underwriting books.

A relevant tangential note should be addressed at this point. It will be evident later in this analysis that the best returns our insurance company can squeeze from its total portfolio seem very low, within a range of about 1.50% to 6.0%. This level of return is not out of line, however, with the returns experienced by the entire property and casualty industry during the decade of the 1990’s. During this period, the industry average return on equity ranged from 8% to less than 0%.⁷ As explained above, and seen in part in the



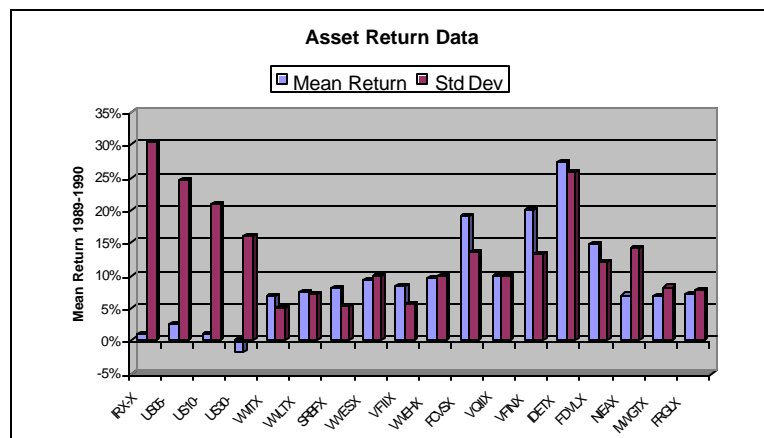
above graph of Industry Profitability, insurers' dismal loss experience and a soft market played a large role in this low level profitability. In fact, the property and casualty insurance industry's returns were so low relative to other industries in the financial sector that insurers lost 26% of their stock market value in 1999 alone.

b. Investment Data

The investment data is the portfolio of investments that will be the subject of the MPT optimum weighting methodology in this paper. The time series data covers the same ten-year period as the underwriting data, and includes descriptive statistics for eighteen investment classes. These classes are proxies for the investments in which an insurer would invest its earned premium. The investments include mutual funds of short, medium, and long duration, debt funds, equity funds, and other investment classes. It is the return on these investments that supports the liabilities (loss payouts, etc.) of the insurer. Negative returns on an investment in a given year may be caused by a lower net investment value at the end of the year than when the year started. The source of this investment return information was MSNBC and Vanguard Inc.

Investment Name	INDEXFAM Short Term Interest Rate Index	INDEXFAM US Treas 5-Yr Yield	INDEXFAM US Treas 10-Yr Yield	INDEXFAM US Treas 30-Yr Yield	Vanguard Tax Exempt Intermediate-Term Inv/42	Vanguard Tax Exempt Long-Term Investor/4	SteinRoe Intermediate Bond	Vanguard Long-Term Corporate Bond Inv/28	Vanguard GNMA/36
Symbol	IRX-X	US05-	US10-	US30-	VWITX	VWLTX	SRBFX	VWESX	VFIIIX
Investment Name	Vanguard High-Yield Corporate/29	Fidelity Convertible Security/308	Vanguard Preferred Stock/38	Vanguard 500 Index Investor	I dex Janus Growth-A	Fidelity Value/039	Eclipse EAFE Index	MFS Global Government-A	Franklin Global Gov Income-A
Symbol	VWEHX	FCVSX	VQIIX	VFINX	IDETX	FDVLX	NIEAX	MWGTX	FRGLX

As seen in the chart below, the average return on these investments for the period was 9.10%, with a standard deviation of 13.28%, a minimum average return of -1.72% on the 30 Year US Bond Index, and a maximum return of 27.36% on the Janus Growth Stock Mutual Fund. The investment data and related calculations are located in Appendix 3.

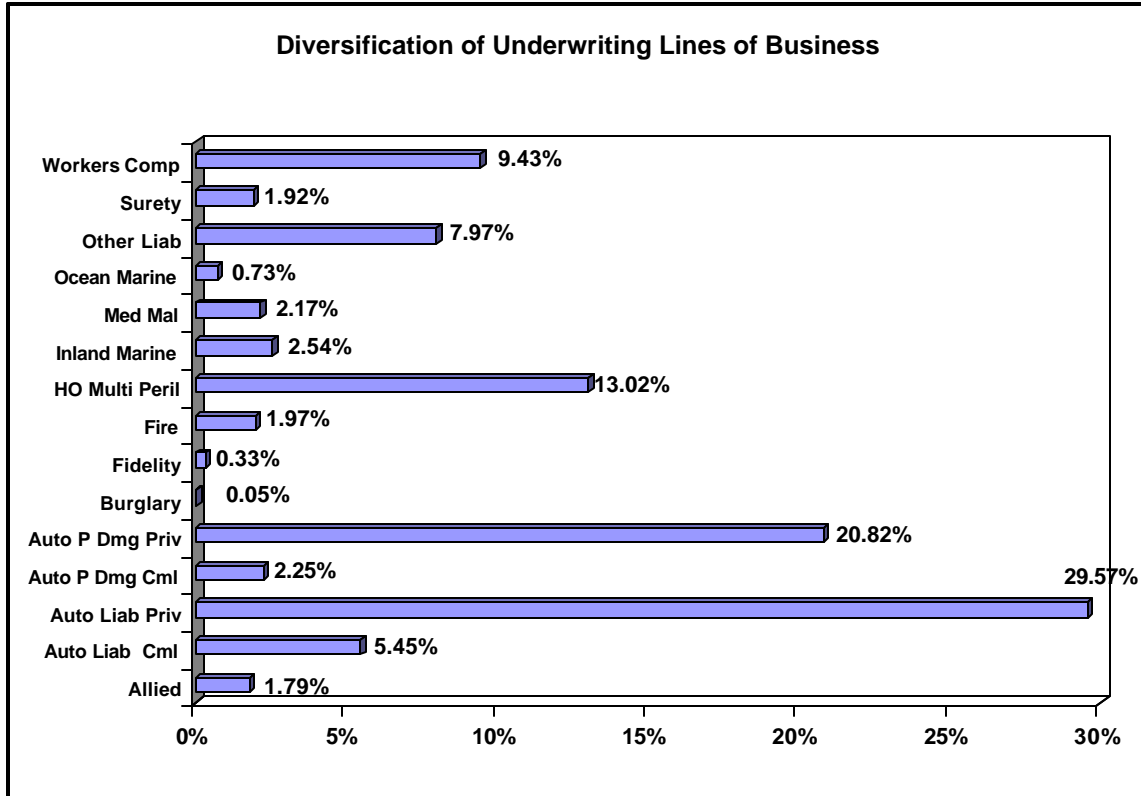


c. Cash Flow Matching

Cash flow matching seeks to match the maturity of the insurer's investments (assets) with the payout patterns of its lines of business or underwriting book (liabilities). The purpose of cash flow matching is to ensure the company has adequate liquidity to pay losses on a timely basis, while at the same time maximizing the return on its investments. The longer the insurer can keep its earned premium invested before it has to pay the sums out in the form of losses and related LAE, the more investment income the insurer earns from that line of business, hence the more profitable the line. Cash flow matching ensures the premium dollars stay invested and earning interest for as long as possible.

The process of matching cash flows begins by considering the size of the insurer's portfolio of investments (investments) relative to the size of its portfolio of liabilities (underwriting book). Between 1998 and 2000, the ratio of investments to liabilities of the average insurer was about 57% to 43%. Of course, the traditional balance sheet formula requires that investments equal liabilities plus owners equity. For the purposes of this mean-variance analysis of investments, however, all that is needed is consideration of investments relative to liabilities. Application of this model to an actual company's data would necessarily take into consideration owners' equity. Given this simplification, investments and liabilities add up to 100% of the insurer's resources being considered. The relative size of the insurer's individual investment and liability portfolios will play a role determining the relative weights of these items in the company's total portfolio, and the mean return and sigma of that overall portfolio.

Next, cash flow matching requires calculation of the relative size of each line of business in the insurer's underwriting book. For the purposes of this analysis, the composition of the book is based on industry aggregate data for 1999, and set forth in the chart below and in Appendix 4.

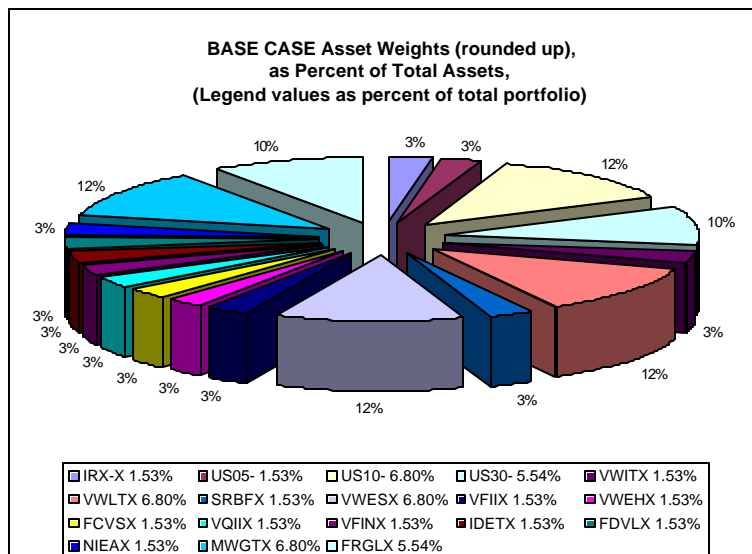


Next, as shown in Appendix 5, each line of business is classified as a Short Tail, Medium Tail, or Long Tail line. The “tail” of a line of business is the amount of time it takes for a majority of claims to be reported and paid under an insurance policy. For purposes of the present analysis, a Short Tail line takes five years or less to develop; a Medium Tail takes six to ten years; and a Long Tail takes eleven or more years to develop.

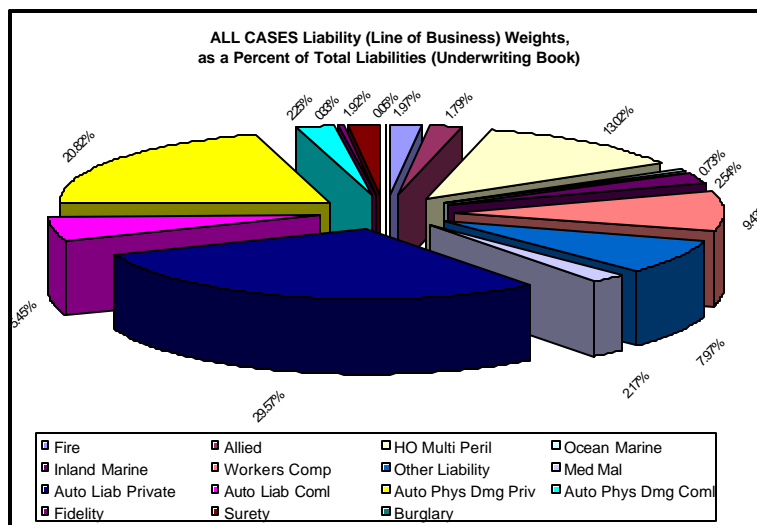
Based on the data, Short Tail lines account for just over 32% of the insurer’s total underwriting book and include the following nine lines of business: Auto Physical Damage Commercial, Auto Physical Damage Private, Burglary, Fidelity, Fire, Inland Marine, Ocean Marine, Surety, and Allied. Medium Tail lines account for about 48% of the insurer’s total underwriting book and include the following three lines of business: Auto Liability Commercial, Auto Liability Private, and Homeowners (HO) Multi Peril. Finally, Long Tail lines account for about 20% of the insurer’s total underwriting book and include the following three lines of business: Other Liability, Medical Malpractice (Med Mal), and Workers Compensation.

The liability or underwriting data can now be matched to investments of the same duration. As demonstrated in Appendix 5, the investments are classified with the same Short, Medium, and Long measures used for liabilities. In our insurer’s case, twelve of the eighteen investment classes are of Short Duration (or tail), four are of

Medium Duration, and two are of Long Duration. With this step, the stage is now set to allocate weights to each investment in the Base Case portfolio: the Short Tail liabilities are matched with Short Duration investments by allowing the total weight allocated to Short Duration investments to total up to a maximum of 32% of the investment portfolio, the same relative size as Short Tail liabilities in the liability portfolio. The initial weight of each of the twelve investments within this class is set arbitrarily at 2.70% (32%/12), i.e., the Short Duration investments are evenly weighted within their duration tranche. The same procedure is applied to the Medium and Long Duration investments, resulting in the four Medium Duration investments accounting for a total 48% of the investment portfolio, evenly weighted relative to each other at 12% each (48%/4), and the two Long Duration investments accounting for a total 20% of the investment portfolio, evenly weighted relative to each other at 10% each (20%/2).



Together, the resulting cash matched investments and liabilities form the total portfolio of the insurer to be considered by the Base Case in our analysis. These portfolios may be represented graphically as shown to the left.



Recall that our ultimate goal is to find the optimal allocation of investment weights. Thus, in subsequent cases, the investment weights to the left will change to provide optimal risk-return trade offs. The weights of each line of business in the insurer's liability (underwriting) portfolio is taken as a given, however, and remains unchanged in all cases.

III. The Model and Its Application

For any portfolio of investments, the expected return of the portfolio is the weighted average of the expected returns on the individual investments, i.e.,

$$E[R_P] = \bar{R}_P = \sum_{i=1}^n w_i \bar{R}_i$$

The variance of a portfolio depends on both the variances and weights of the individual investments in the portfolio, as well as the covariance between them, i.e.,

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_{AB}$$

Once we calculate portfolio variance, we can simply take its square root to find the portfolio's standard deviation.

The MPT model utilized herein is a simple quadratic programming statement set forth in an Excel worksheet using the Solver add-in. The statement sets the optimal weight for each investment in the insurer's investment portfolio with an eye to minimizing the portfolio's variance while maximizing its return. The model is also subject to the following constraints: the sum of the optimized weights must add up to 100%; each weight must be less than or equal to 100% and greater than or equal to 0% (i.e., no short sales); and for cash matching purposes, the total weight of short duration investments must sum to the industry average for the short duration tranche (32.39%), the total weight of medium duration investments must sum to the industry average for the medium duration tranche (48.04%), and the total weight of long duration investments must sum to the industry average for the long duration tranche (19.57%).

With this model now set, the analysis turns to calculating the mean return and sigma of the insurer's Base Case portfolio. As explained above, this is a simple portfolio where each constituent investment is evenly weighted within its duration tranche. The subsequent cases involve applications of MPT with increasingly more information about the insurer's total portfolio to determine whether the company's overall return can be increased.

a. Base Case

This case assumes the insurer uses a very simple allocation system. First, except for cash flow matching, the insurer considers investments and liabilities (underwriting lines of business) separately. The mean-variance characteristics of its underwriting

book are not considered in allocating investments in its investment portfolio. The diversification (weights) of the lines of business in the insurer's underwriting book is simply the industry average and does not change. The weights of the investments on the investment side are equally weighted by duration tranche (see the pie charts above). Once established, the weights of each constituent element of the investment and liability side are converted to their weight as part of the total portfolio: recall that the (1999 average) ratio of investments to liabilities of the average insurer is about 57% to 43%. Each element of the insurer's overall portfolio is converted using these ratios. Thus, for example, the insurer invests 2.70% of total investments in short-term interest index instruments; this converts to 1.53% ($2.70\% * 57\%$) of the insurer's total portfolio of investments and liabilities.

With this information, we can apply the above formulas to determine the mean return and standard deviation of the Base Case portfolio. These calculations are in Appendices 6 and 7:

	Base Case
Std Dev	2.11%
Mean	1.35%

b. Case 2

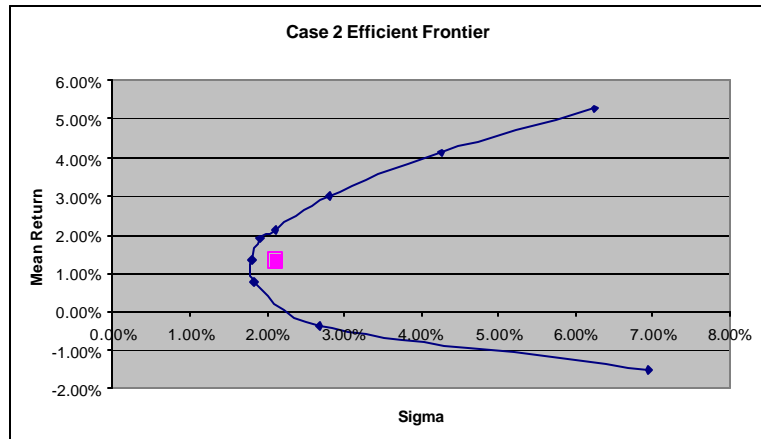
This is the first case in which this analysis applies MPT. The application is limited, however, to the investment side of the insurer's balance sheet. This case also ignores mean-variance characteristics of its underwriting book in optimizing the weights on the investment side, though the weights and by-line returns on the liability side are considered in calculating the portfolio return and standard deviation. This application of MPT only to the investment side (not liabilities) might be considered the traditional application of the methodology. Cash matching continues to apply in this case (as in all cases in this analysis). The resulting descriptive statistics are as follows:

	Base Case	Case 2
Std Dev	2.11%	1.80%
Mean (constant)	1.35%	1.35%
Std Dev (constant)	2.11%	2.11%
Mean	1.35%	2.14%

Note the improvement in the mean-variance characteristics of the insurer's overall portfolio. By applying MPT (considering only investment mean-variance), the insurer is able to obtain from its total portfolio the *same* mean return as the Base Case but with a *lower* standard deviation. Likewise, in Case 2, the insurer is able to obtain from

its total portfolio the *same* standard deviation as the Base Case but with a *higher* mean return.

This improvement in the insurer’s overall performance is also seen in a graph of the efficient frontier of Case 2. In the graph to the right, the Base Case is plotted as the point in pink color. The efficient frontier of Case 2 is to the left and higher than the Base Case, indicating the ability of the insurer to obtain a higher mean return with a commensurately lower standard deviation. The calculations for Case 2 are located in Appendices 8 and 9.

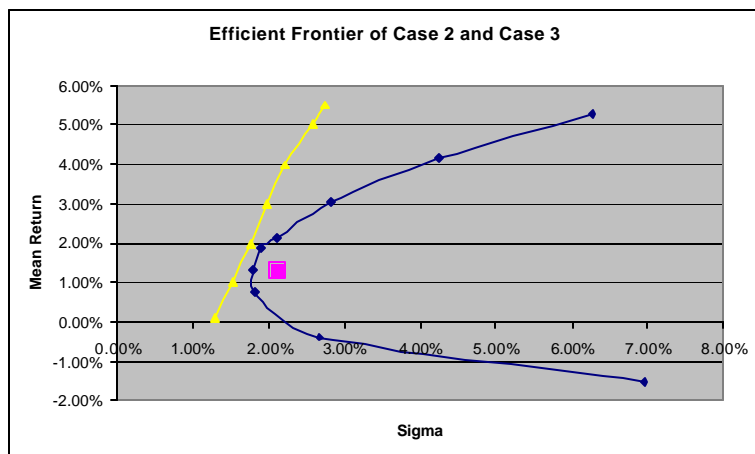


c. Case 3

Case 3 uses the same parameters and MPT model as Case 2, except now the analysis adds consideration of the characteristics of the insurer’s underwriting book (the return on liabilities) in order to calculate the optimum weights of the insurer’s investments in its investment portfolio. As in the prior cases, the diversification of the underwriting book remains unchanged. As demonstrated below and in Appendix 10, the result of applying the MPT model is still further improvement of the results of the total portfolio:

	Base Case	Case 2	Case 3
Std Dev	2.11%	1.80%	1.62%
Mean (constant)	1.35%	1.35%	1.35%
Std Dev (constant)	2.11%	2.11%	2.11%
Mean	1.35%	2.14%	3.65%

Likewise, the new efficient frontier of Case 3 (in yellow below) is even further to the left of Case 2 and the Base Case, indicating the insurer’s ability to continue to lower total portfolio standard deviation yet improve return. Note too the improved slope of the efficient frontier of Case 3 over Case 2: in Case 2, moving along the efficient frontier from left to right increases return from 1.35% to about 5.50% with a related increase in sigma from 1.80% to about 6.30%; the same move on the efficient frontier



of Case 3 increases the mean return to the same level as Case 2, but with a smaller range of sigma, from about 1.50% to about 2.50%:

Case 3 therefore represents a marked improvement over the Base Case and Case 2. The determining factor in this improvement was simply to consider the insurer's total

portfolio – investments as well as underwriting book – in applying MPT. This is a non-traditional application of MPT. The methodology adapted the weighting of the investments to immensely mitigate the volatility of the underwriting book without sacrificing overall portfolio return.

d. Case 4

This analysis has concentrated on improving the returns on the insurer's investments by applying the MPT, mean-variance optimization technique. In Case 4, our insurance company seeks to improve further its results by purchasing reinsurance to seek improvements in the return on its underwriting book of business (its liabilities).

Pursuant to a reinsurance agreement, the company seeks a counterparty (another insurance company) to assume a portion of the insurance obligations – its liability – in its underwriting book. In return, the insurance company pays the reinsurer a commensurate portion of the premium it collected for the underlying insurance business. Thus, our insurance company “cedes” risk and premium to another company that “assumes” the business under the reinsurance agreement. The assuming insurer (the reinsurer) will also pay the ceding insurer a ceding commission to reimburse a portion of the costs to underwrite the direct business, taxes, and related costs. The expectation is the reinsurance will improve the return on the underwriting book of the ceding insurer by reducing the size and volatility of its liabilities.

For the purposes of this analysis, a reinsurer furnishes proportional reinsurance to cover all the ceding insurer's lines of business, with the reinsurer assuming 30% of the direct insurer's underwriting book, in return for 30% of the premium related to this direct business. The reinsurer will pay the direct insurer a ceding commission of 25% of the ceded premium. In other words, the reinsurance reduces the ceding insurer's

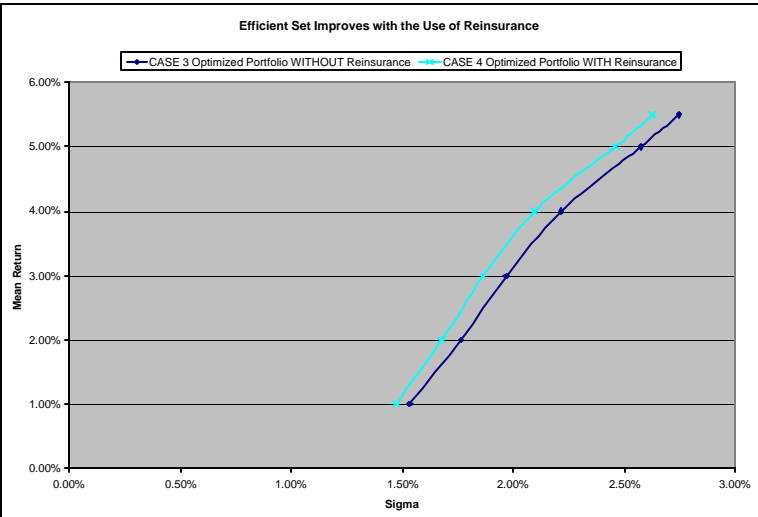
liabilities by 30%, along with a reduction of 22.5% of its premium revenue (pay 30% of premium to reinsurer, receive 25% * 30% or 7.5% as ceding commission). The reinsurance therefore ultimately changes the ceding insurer’s underwriting return. The by-line calculations of these underwriting returns are in Appendix 11. The salutary effects on the ceding insurer’s underwriting returns are demonstrated in Appendix 12.

With the revised underwriting (liability) data in hand, the insurer can seek additional improvements in its total portfolio by applying MPT. As in Case 3, the application of MPT in Case 4 takes into account the characteristics of both the insurer’s investments and liabilities, though it optimizes the allocation of its investments only. All other cash matching and other model constraints apply. As demonstrated below and in Appendix 13, the result is still further improvement of the results of the total portfolio:

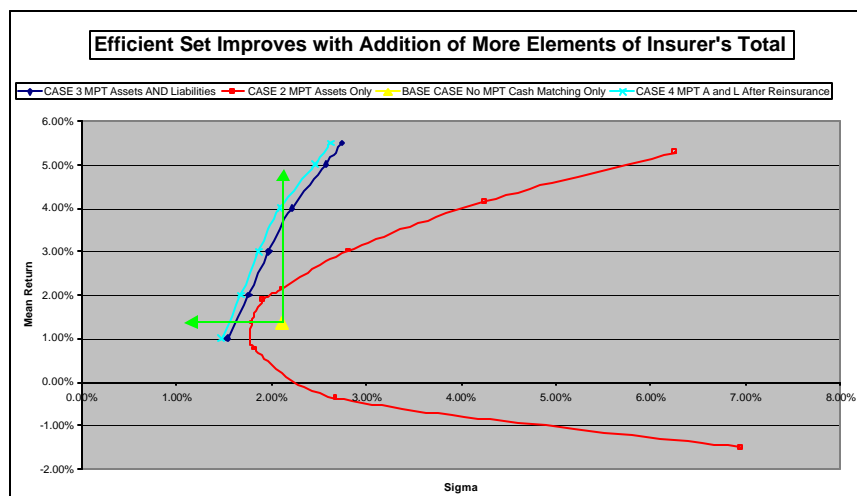
	Base Case	Case 2	Case 3	Case 4
Std Dev	2.11%	1.80%	1.62%	1.55%
Mean (constant)	1.35%	1.35%	1.35%	1.35%
Std Dev (constant)	2.11%	2.11%	2.11%	2.11%
Mean	1.35%	2.14%	3.65%	4.04%

The positive effect of reinsurance and MPT on the return in the insurer’s total portfolio is seen graphically by comparing the results of Cases 3 and 4. In the graph below, the insurer’s Case 4 efficient frontier has moved to the left of the Case 3, indicating higher Case 4 returns for the same portfolio sigmas of Case 3, and lower Case 4 sigmas for the same portfolio returns of Case 3.

The slope of Case 4’s efficient frontier has not changed from Case 3. This is likely because the effect of the reinsurance was an even, across the board reduction of the same liabilities used in Case 3. Thus, the reinsurance reduced the volatility of the ceding insurer’s underwriting book and improved its return, but it did so in a uniform manner resulting in no change from Case 3 in the slope of the efficient frontier.

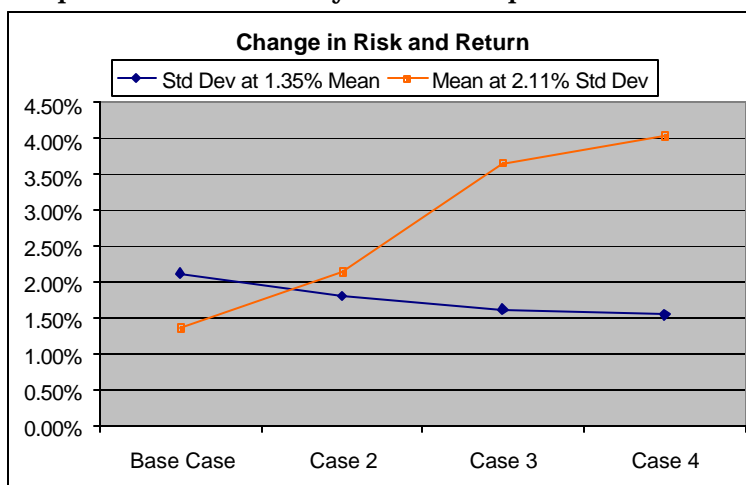


IV. The Results



Together, application of MPT and the use of reinsurance have valuable, positive effects on the overall return and volatility in the investments and underwriting book of an insurance company. Starting with the Base Case and moving up through the efficient

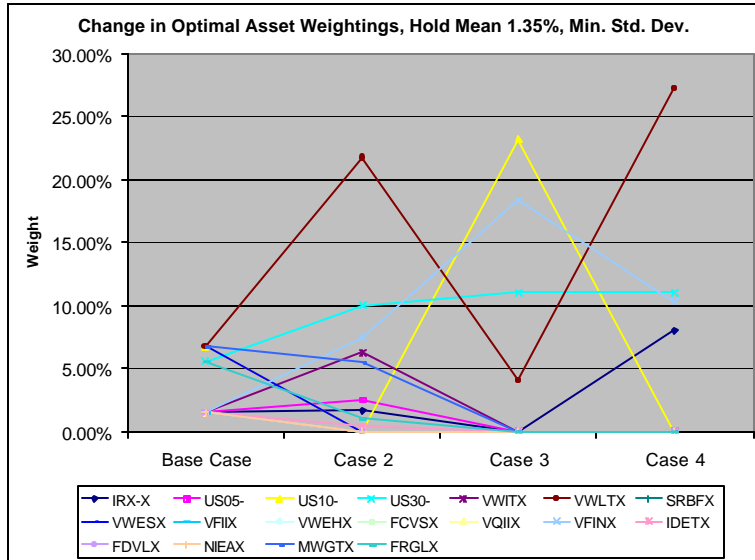
frontiers of the subsequent cases shows for the same standard deviation of 2.11%, the ceding insurer can improve its total portfolio returns by 269 basis points or 200% – from 1.35% to just over 4.0% in the simple tests herein – depending upon how much portfolio information its MPT analysis employs, and whether the company uses reinsurance. Likewise, starting with the Base Case and moving to the left through the efficient frontiers of the other cases, the insurer can maintain the same overall return (1.35%) yet lower its total portfolio volatility (standard deviation) by 56 points or 27% – from 2.11% to 1.55% – depending upon its use of MPT and reinsurance.



Note that an insurer can improve its lot further than the cases tested here by abandoning the Base Case and simply choosing the optimal portfolio best for its interests on one of the efficient frontiers. For example, the insurer may choose the Case 4 efficient frontier and reinsurance. The company may opt to take on more risk than the 2.11% risk of the Base Case, hence perceive a return higher than the 1.35% of the Base Case, e.g., the insurer may seek an overall return of 5.5% because it believes it can manage the increased risk of 2.63% experienced at this level of return.

a. Note on Optimal Diversification of Investments

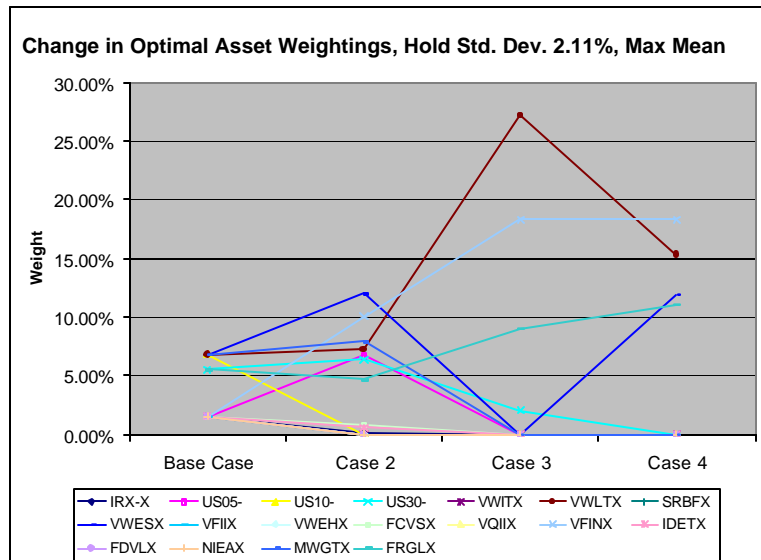
As demonstrated in the graphs below, the MPT optimization process reallocated the investments of the insurance company in order to maximize return and reduce risk. It is notable, however, that in fulfilling this goal, the MPT process actually utilizes a *lower*



number of investments in the efficient portfolios as it takes into consideration more elements of the insurer's overall portfolio. That is, the Base Case assumes equal weight among *all* eighteen investments. MPT is applied in Case 2, considering only investment characteristics (ignoring liabilities). In this case, the methodology utilizes only nine of the eighteen investments. Case 3 and Case 4 consider the characteristics

of both investments and the underwriting book, and Case 4 adds reinsurance. In applying MPT in these latter two cases, the methodology needs only four of the eighteen investments in order to generate the efficient portfolio. This utilization of a low number of investments is the same at all levels of returns and standard deviation.

The model likely uses fewer of the investments for two reasons. First, the algorithm used by Excel Solver may simply be economical in its search to satisfy all conditions in the model, stopping its operation once it finds the right combination among the first several investments that satisfy the objective function, even though it has not utilized all the investments it has to select from. One could add a constraint to the model, if desired, forcing Solver to use at least some percentage of all investments. Second, in Cases 3 and 4 (where the model utilized the fewest investments), the model considers the characteristics of



both investments and the underwriting book, with the allocation among lines in the underwriting book taken as given. To the extent the lines of business *in the underwriting book* help satisfy the objective function, Solver apparently needs to consider only a few of the investments *in the investment portfolio* order to reach its goal. In Case 2, the MPT application considered *only* investments, and ignored the underwriting book, hence Solver had to find all the elements necessary to satisfy the objective function from the investments alone, therefore it utilized more of the investments.

Does this point on diversification say anything about MPT or optimal portfolios in the context of insurance? This issue might be seen as simply a characteristic of the Solver algorithm, however, it may also be an indication that an insurer in reality needs few investment classes per duration tranche to support payment of losses and related expenses from its underwriting book. Of course, because MPT is, as noted, a single period model, the portfolio of these few investment classes upon which an insurer might rely needs to be rebalanced regularly.

b. The Effect of Reinsurance

As explained earlier, some commentators have suggested an insurer may, in the face of unattractive reinsurance arrangements, be better off securitizing its insurance risks within its optimized investment portfolio, rather than purchasing reinsurance. The flaw in this rationale is the unusual proportional reinsurance arrangements it contemplates. The present analysis demonstrated customary proportional reinsurance arrangements have a positive effect on an insurance company's overall return.

Avoiding reinsurance and relying entirely on a good investment strategy would also have the insurer place a very large, risky bet on the positive performance of investments to mitigate a possibly negative performance of its liabilities. The risk of this strategy is especially apparent where an unforeseen catastrophe inflates liabilities to a size dwarfing an excellent investment portfolio, or a market bubble unexpectedly bursts, resulting in the bet on investments being a bad one. Using reinsurance, on the other hand, is a "sure thing" because it brings foreseeability into the equation, as the insurer knows the amount of risk it retains and how much it cedes to the reinsurer.

V. Conclusion

The principal goal of this paper was to demonstrate the utility of modern portfolio theory in the investment allocation decisions of an insurance company. The paper fulfilled this goal by considering an insurer to be simply one portfolio of assets and liabilities, including reinsurance. The paper's all-inclusive yet simple model succeeded in presenting how an insurer might lower the overall risk while increasing its overall

return. With the appropriate modifications, there is little doubt an actual insurance company can adapt the model presented herein to improve its results.

ENDNOTES

¹ Adapted from Mehndi Pirbhai ,“Portfolio Optimisation: The rise of Markowitz Mean-Variance Models And Beyond” (2001).

² Joan Lamm-Tennant, “The NAIC Model Investment Law: Implications for Optimal Capital Allocation Decisions” (1996).

³ Adapted from “The Property and Casualty Insurance Industry,” Coopers and Lybrand and the Task Force on the Future of the Canadian Financial Services Sector (1998).

⁴ Reinsurance comes in many different forms, however, one can generalize between two principal types: proportional reinsurance and excess of loss reinsurance. Proportional reinsurance is also known as pro rata reinsurance (and quota share and participating reinsurance), where an assuming insurer indemnifies a ceding company for a fixed percent of loss on each risk covered in the contract in consideration of the same percentage of the premium paid to the ceding company. Excess of loss reinsurance is a form of reinsurance that, subject to a specified limit, indemnifies the ceding company against the amount of loss in excess of a specified retention. It includes various types of reinsurance, such as catastrophe reinsurance, per risk reinsurance, per occurrence reinsurance and aggregate excess of loss reinsurance.

⁵ James E. Bachman and Joan Lamm-Tennant, “Reinsurance and Managing Company Risk,” Best’s Review P/C, (Nov. 1995).

Ten Most Costly Catastrophes, United States		Insured loss (\$ millions)	
Date	Peril	Dollars when occurred	In 2000 dollars (1)
Aug.1992	Hurricane Andrew	\$15,500.0	\$19,024.2
Jan.1994	Northridge, California earthquake	12,500.0	14,524.3
Sep.1989	Hurricane Hugo	4,195.0	5,825.6
Sep.1998	Hurricane Georges	2,900.0	3,063.7
Oct.1995	Hurricane Opal	2,100.0	2,372.6
Sep.1999	Hurricane Floyd	1,960.0	2,025.9
Mar.1993	20-state winter storm	1,750.0	2,085.5
Oct.1991	Oakland, California fire	1,700.0	2,149.3
Sep.1992	Hurricane Iniki	1,600.0	1,963.8
Sep.1996	Hurricane Fran	1,600.0	1,756.0

(1) Adjusted to 2000 dollars by the Insurance Information Institute.

Source: Insurance Services Office, Inc., Insurance Information Institute.

⁷ Source: Conning & Company.